

Composite Particles in the Theory of Quantum Hall Effect

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The formation of composite particles in the electron liquid under QHE conditions discussed by Jain in generalizing Laughlins many-particle state is considered by using a model for two-dimensional guiding center configurations. Describing the self-consistent field of electron repulsion by a negative parabolic potential on effective centers and an inter-center amount we show that with increasing magnetic field the ground state of so-called primary composite particles $\nu = \frac{1}{q}$, $q = 1, 3, 5, \dots$, is given for higher negative quantum numbers of the total angular momentum. By clustering of primary composite particles due to absorption or emission of flux quanta we explain phenomenologically the quasi-particle structure behind the series of relevant filling factors $\nu = \frac{p}{q}$, $p = 1, 2, 3, \dots$.

Our considerations show that the complicate interplay of electron-magnetic field and electron-electron interactions in QHE systems may be understood in terms of adding flux quanta Φ_0 to charges e and binding of charges by flux quanta.

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I. INTRODUCTION

Since the experimental discovery of the integral and fractional quantum Hall effect (IQHE, FQHE) [1,2] this macroscopic quantum phenomenon is a real challenge in particular for theoretical and mathematical physicists. Under QHE conditions determined by interacting quasi-twodimensional charges of high mobility at very low temperatures in presence of a strong perpendicular magnetic field B and a disorder potential U electron correlation has been discussed by using several approaches.

In 1983 Laughlin [3] introduced an antisymmetric many-particle wave function for the fermionic system characterising the ground state of the correlated quantum liquid by the odd numbered parameter $m = 1, 3, 5, \dots$. Laughlins trial wave function may be written in the form

$$\chi_{\frac{1}{m}} = \prod_{\substack{j,k \\ (j < k)}} (z_j - z_k)^{m-1} \chi_1 \quad (1)$$

with

$$\chi_1 = \prod_{\substack{j,k \\ (j < k)}} (z_j - z_k) \exp\left(-\frac{1}{4} \sum_{i=1}^{N_e} |z_i|^2\right). \quad (2)$$

Here $z_j = x_j + iy_j$ denotes the dimensionless electron positions in units of the magnetic length $l_0 = \sqrt{\frac{\hbar}{eB}}$, N_e is the total electron number. In this approach, interaction has been taken into account for spin-polarized electrons in the lowest Landau level $N = 0$. The many-particle state (1) contains m fluxes per electron. It is an exact solution for harmonic potentials [4], but an approximation for totally included Coulomb interaction.

A generalization of Laughlins construction was given in 1989 by Jain [5] who proposed a FQHE state of the form

$$\chi_{\nu} = \prod_{\substack{j,k \\ (j < k)}} (z_j - z_k)^{m-1} \chi_p \quad (3)$$

where χ_p , $p = 1, 2, 3, \dots$ denotes an unknown antisymmetric function describing the p -th IQHE state. Then one composes the number of fluxes per electron for the p -th Landau level $\frac{1}{p}$ with the even number $(m-1)$ of fluxes in analogy to expression (1). By this way we obtain all relevant filling factors

$$\nu = \frac{p}{(m-1)p \pm 1}. \quad (4)$$

In (4) the number p can be interpreted as Landau level index N in the sense $p = N + 1$, $N = 0, 1, 2, \dots$, and electron number as well. The quantized filling factor (4) $\nu = \frac{p}{q} = \frac{\hbar \mathcal{N}_s}{eB}$ with the $2d$ -electron concentration \mathcal{N}_s , describes composite particles (CP) which connect p charges e with q flux quanta $\Phi_0 = \frac{h}{e}$.

Generally, correlation in the considered many-particle system may be investigated also by topological and field theoretical methods which leads to an interpretation of q and p as winding numbers of the magnetic field and the self-consistent gauge field, respectively [6]. Then, interaction and structure formation are reflected by the linking number $\nu = \frac{p}{q}$.

The actual quasi-particle picture for the FQHE ground state makes use of attaching an even or odd number of fluxes to a charge leading to composite fermions (CF) or bosons (CB), respectively. The statistics transmutes between these composite particles.

In this paper we give a local approach to the QHE quasi-particle picture using a simplified description by an effective potential on the guiding centers \vec{R}_i which are surrounded by appropriate one-electron states (n, m) . Because of the famous works of Fock [7] and Landau [8], it is well-known that the Schrödinger equation for a single electron in a constant magnetic field leading to potentials proportional to r^2 and $\frac{1}{r^2}$, has exact solutions. In dependence on boundary conditions and calibration of the vector potential one finds Landau functions or radial symmetric one-particle states. In [6] we started with the latter to investigate the dynamical electron-electron interaction including spin structures by topological techniques. In this paper the relevant electron-electron interaction is modelled by a mean-field potential and a cluster construction which aims at a detailed illustration of the quasi-particle structure observed in QHE experiments [9,10].

II. PRIMARY COMPOSITE PARTICLES

As the starting point we consider spinless electrons described by Schrödinger's equation with the effective Hamiltonian

$$H = H_{eB} + H_{eV} + H_{eU} \quad (5)$$

where B , V and U denote the homogeneous magnetic field, a selfconsistent electric potential (including partially ee-interaction) and the disorder potential, respectively. Later on we take into account the Fermi or Bose statistics which result from electron correlation beyond the mean field description. The eigenvalue equation for H_{eB} in symmetric gauge for the vector potential $A_\phi = \frac{1}{2}Br$ has the form

$$\left\{ -\frac{\hbar^2}{2M} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right] - \frac{i\hbar\omega_c}{2} \frac{\partial}{\partial \phi} + \frac{M\omega_c^2}{8} r^2 \right\} \Psi(r, \phi) = E\Psi(r, \phi) \quad (6)$$

with the cyclotron frequency $\omega_c = \frac{eB}{M}$. Equation (6) is exactly solved by

$$\Psi_{nm} = \frac{1}{l_0} \tilde{R}_{nm}(r) \frac{1}{\sqrt{2\pi}} e^{im\phi} \quad (7)$$

and

$$E_{nm} = \hbar\omega_c \left[n + \frac{|m|+1}{2} + \frac{m}{2} \right] = \hbar\omega_c \left(N + \frac{1}{2} \right) \quad (8)$$

with the radial quantum number $n = 0, 1, 2, \dots$ and the magnetic quantum number $m = -\gamma, \dots, 0, \dots, N$; γ denotes the degree of the degeneracy of the Landau level N for the finite quantum system. The well-known radial functions $\tilde{R}_{nm}(r)$ containing polynomial and exponential factors are related to the guiding center at $\vec{R} = 0$. In the following we consider the many-particle system in the parameter space of guiding centers forming a $2d$ -configuration $\{\vec{R}_i\}$.

In a first step the one-particle Hamiltonian H_{eV} has to be determined. For an effective center in the electron layer of thickness l_d the repulsive ee-interaction between the states (7) should be modeled by the potential $V(r)$ of an average charge e on a homogeneous cylinder with radius $l_r(B)$. Solving Poisson's equation for the center at $\vec{R} = 0$

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{d}{dr} \right) V(r) = -\frac{e}{\epsilon_0 \pi l_r^2 l_d} \quad (9)$$

one obtains the solution

$$V(r) = -\frac{e}{4\pi\epsilon_0 l_d} \begin{cases} \left(\frac{r}{l_r}\right)^2 & \text{for } r < l_r \\ (2 \ln\left(\frac{r}{l_r}\right) + 1) & \text{for } r > l_r \end{cases} \quad (10)$$

which approximates magnetic field dependent ee-interaction in the q2d-system decomposed in a negative quadratic part for $r < l_r(B)$ and a logarithmic long distance part. Including the effective Hamiltonian

$$H_{eV} = -\frac{M}{2}\omega_0^2 r^2 \quad (11)$$

with the parameter

$$\omega_0^2(B) = \frac{e^2}{2\pi\epsilon_0 M l_d l_r^2(B)} \quad (12)$$

representing the one-particle approximation in Schrödingers equation (6) we can note that the structure of wave functions (7) remains unchanged whereas the degeneracy in (8) is lifted according to

$$\tilde{E}_{nm} = \hbar\tilde{\omega} \left[n + \frac{|m|+1}{2} + \frac{m}{2} \frac{\omega_c}{\tilde{\omega}} \right] \quad (13)$$

with

$$\tilde{\omega}^2 = \omega_c^2 - 4\omega_0^2 = \frac{e^2 B}{M^2} (B - B_c) . \quad (14)$$

Equation (14) holds for $B > B_c \sim 1T$ where we used the rough estimation $l_r(B) \simeq 10^2 \dots 10^3 \text{nm}$ and $l_d \simeq 10 \text{nm}$. It determines a smaller renormalized energy scale $\hbar\tilde{\omega} < \hbar\omega_c$ for the quasi-particles in comparison to the bare particles. In case of vanishing ee-interaction one has $\omega_0 = 0$, $\tilde{\omega} = \omega_c$ and the energy e.g. of the degenerated, lowest Landau level $N = 0$ is $E_{00} = \frac{1}{2}\hbar\omega_c$ for $n = 0$, $m = 0$ and all negative values of m .

Looking for the intriguing interplay between ee- and eB-interactions in the considered system we realize that the flux quantum number $|m|$ includes spin structures according to $|m| = 2s + 1$ where $s = \frac{1}{2}, \frac{3}{2}, \dots \rightarrow |m| = 2, 4, \dots$ and $s = 0, 1, 2, \dots \rightarrow |m| = 1, 3, 5, \dots$ reflect fermionic and bosonic quasi-particle states (7), respectively.

The splitted energy levels (13) for $n = 0$ and $m = 0, -1, -2, -3, -4, -5, \dots$ are given in table I. Table I shows, that with increasing magnetic field the ground state will be determined by higher $|m|$ and the relative influence of ee-interaction on the quasi-particle energy decreases. The splitted energy levels are restricted by the condition

$$\tilde{E}_{nm} > 0 \quad (15)$$

which means that only a definite number of levels \tilde{E}_{0m} with negative m takes place in the confining potential which is caused by the magnetic field. If we fix for example the magnetic field at $B = B^*$ and the parameter value $\omega_0(B^*)$ at

$$\left(\frac{\omega_c}{\tilde{\omega}}\right)^* = \frac{5}{4} \quad (16)$$

then it holds that all levels down to

$$\tilde{E}_{0,-3} = \hbar\tilde{\omega}^* \left(2 - \frac{15}{8}\right) = \frac{1}{8}\hbar\tilde{\omega}^* > 0 \quad (17a)$$

are allowed but

$$\tilde{E}_{0,-4} = \hbar\tilde{\omega}^* \left(\frac{5}{2} - \frac{5}{2}\right) = 0 \quad (17b)$$

and

$$\tilde{E}_{0,-5} = \hbar\tilde{\omega}^* \left(3 - \frac{25}{8}\right) = -\frac{1}{8}\hbar\tilde{\omega}^* < 0 \quad (17c)$$

are forbidden. For a higher magnetic field B^* with e.g. $(\frac{\omega_c}{\omega})^* = \frac{7}{6}$ the levels $\tilde{E}_{0,-4}$ and $\tilde{E}_{0,-5}$ are allowed but $\tilde{E}_{0,-6}$ is forbidden. Thus there exists a magnetic field dependent ground state with a maximum value $|m|_{max}$ that fulfills condition (15) for the ideal system without disorder.

In order to discuss the important role of disorder we introduce the new parameter τ_{coll} being the time between two collisions of the quasi-particle with the disorder potential U . In the real system with randomly distributed impurities besides (15) the cyclotron condition

$$\tilde{\omega}\tau_{coll} > 1 \quad (18)$$

has to be fulfilled. For a certain degree of disorder τ_{coll} has a definite value τ^* leading to

$$\tilde{\omega}\tau^* = |m|_{coll} \quad (19)$$

where $|m|_{coll} > 1$ is generally an irrational number which must be replaced by the next lower integer. Therefore one has to distinguish between the cases

$$|m|_{coll} > |m|_{max} \quad (20a)$$

and

$$|m|_{coll} < |m|_{max} , \quad (20b)$$

where $|m|_{max}$ would be allowed in the ideal system without disorder where only condition (15) must be fulfilled. Obviously the real quasi-particle ground state has to be characterized by the optimum value $|m|_{opt}$ that is the lower integer value of the two numbers $|m|_{max}$ and $|m|_{coll}$

$$|m|_{opt} = \min(|m|_{max}, |m|_{coll}) . \quad (21)$$

Resulting from our consideration we identify the module of the relevant negative quantum number m with the number of fluxes q belonging to one charge for a composite particle $\nu = \frac{1}{q}$. In case of an odd number of fluxes attached to the reference state $|0, 0\rangle$ one has Laughlin quasi-particles in the sense of composite bosons. The index ν is used here for the quasi-particle ground state and the filling factor as well. With $\nu = 1, \frac{1}{3}, \frac{1}{5}, \dots$ we denote so-called primary composite particles.

III. CLUSTERED COMPOSITE PARTICLES

Here we remember that the one-particle Hamiltonian (5) with the effective potential part (11) accounts for ee-interaction only approximately on short distances. There remains an unknown rest of Coulomb interaction between primary CP. This correlation part should be responsible for cluster formation in the relevant quantum liquid leading to Jain quasi-particles. These N -body interactions may be discussed in a phenomenological picture employing the transfer of flux quanta from the magnetic field B to the CP or in opposite direction.

Starting from the bosonic primary CP $\nu = 1$ which combines one charge and one flux

$$\nu = \frac{p}{q} = \frac{1}{1} = 1$$

a cluster of $p = 2, 3, 4, \dots$ CPs is generated by the scheme

$$\begin{aligned} \frac{1}{1} \oplus \frac{1}{1} \oplus \frac{0}{1} &\longrightarrow \frac{2}{3} \\ \frac{1}{1} \oplus \frac{1}{1} \oplus \frac{1}{1} \oplus \frac{0}{2} &\longrightarrow \frac{3}{5} \\ \dots &\longrightarrow \left(\frac{1}{2}\right)^+ . \end{aligned} \quad (22)$$

Here the symbol \oplus stands for clustering and $\frac{0}{1}, \frac{0}{2}, \dots$ denotes binding flux quanta changing from the B field to the cluster.

If we start from other primary CP it holds

$$\begin{aligned}
\nu &= \frac{1}{3} \\
\frac{1}{3} \oplus \frac{1}{3} \ominus \frac{0}{1} &\longrightarrow \frac{2}{5} \\
\frac{1}{3} \oplus \frac{1}{3} \oplus \frac{1}{3} \ominus \frac{0}{2} &\longrightarrow \frac{3}{7} \\
\ldots &\longrightarrow \left(\frac{1}{2}\right)^-
\end{aligned} \tag{23}$$

where the symbol \ominus indicates the change of flux quanta from the CP into the acting field B . Consequently one finds

$$\begin{aligned}
\frac{1}{3} \oplus \frac{1}{3} \oplus \frac{0}{1} &\longrightarrow \frac{2}{7} \\
\frac{1}{3} \oplus \frac{1}{3} \oplus \frac{1}{3} \oplus \frac{0}{2} &\longrightarrow \frac{3}{11} \\
\ldots &\longrightarrow \left(\frac{1}{4}\right)^+
\end{aligned} \tag{24}$$

and furthermore

$$\begin{aligned}
\nu &= \frac{1}{5} \\
\frac{1}{5} \oplus \frac{1}{5} \ominus \frac{0}{1} &\longrightarrow \frac{2}{9} \\
\frac{1}{5} \oplus \frac{1}{5} \oplus \frac{1}{5} \ominus \frac{0}{2} &\longrightarrow \frac{3}{13} \\
\ldots &\longrightarrow \left(\frac{1}{4}\right)^-
\end{aligned} \tag{25}$$

and

$$\begin{aligned}
\frac{1}{5} \oplus \frac{1}{5} \oplus \frac{0}{1} &\longrightarrow \frac{2}{11} \\
\frac{1}{5} \oplus \frac{1}{5} \oplus \frac{1}{5} \oplus \frac{0}{2} &\longrightarrow \frac{3}{17} \\
\ldots &\longrightarrow \left(\frac{1}{6}\right)^+
\end{aligned} \tag{26}$$

From the scheme (22)-(26) and table I one sees that in dependence on B ground state energies \tilde{E}_ν for $\nu = \frac{p}{q}$ are lowered by absorption of flux quanta whereas emission of fluxes leads to higher \tilde{E}_ν . The infinite series of ν converge from both sides to the limits $(\frac{1}{2}), (\frac{1}{4}), (\frac{1}{6}), \dots$ describing composite fermions.

In our picture the IQHE is naturally included. From

$$\nu = \frac{p}{q} = \frac{1}{1} = 1$$

one has

$$\begin{aligned}
\frac{1}{1} \oplus \frac{1}{1} \ominus \frac{0}{1} &\longrightarrow \frac{2}{1} \\
\frac{1}{1} \oplus \frac{1}{1} \oplus \frac{1}{1} \ominus \frac{0}{2} &\longrightarrow \frac{3}{1} \\
\ldots &\longrightarrow N \rightarrow \infty
\end{aligned} \tag{27}$$

which demonstrates the well-known one-particle behaviour of electrons on fully occupied Landau levels. Fractionally values in the region $1 < \nu < 2$ are generated by

$$\begin{aligned}
1 + \left(\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \dots \right) &\longrightarrow \left(\frac{3}{2} \right)^- \\
1 + \left(\frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \dots \right) &\longrightarrow \left(\frac{3}{2} \right)^+
\end{aligned} \tag{28}$$

in correspondence to fully and partially filled Landau levels.

Finally we remark that the cluster construction (22)-(28) claims primary composite bosons $\nu = \frac{1}{1}, \frac{1}{3}, \frac{1}{5}, \dots$ which permits a deeper insight into the condensation to fermionic QHE states e.g. the state $\nu = \frac{1}{2}$ of the half filled lowest Landau level.

IV. QUALITATIVE DISCUSSION OF THE EE-INTERACTION

After the remarkable coincidence between the phenomenological model used in the previous sections and the observed conductivity fractions [9,10], we are trying to motivate the relevant interaction term (11) in the effective Hamiltonian (5). From a general point of view we describe a plane of interacting electrons with fluctuating geometry. The macroscopic Hall conductivity as an expectation value of the corresponding microscopic current-current correlator is a topological invariant [6]. Now the idea is to encode the topological information of the system into its geometrical structure in order to determine an effective Hamiltonian containing the main part of interaction.

For that purpose we consider a system of N electrons under QHE conditions on a surface \mathbb{C} . The construction of the configuration space is given in the following way. At first we keep in mind that every single-particle wave function admits a support which means an area with non-zero value. The configuration of one fermion is simply the whole surface \mathbb{C} . In case of two particles we fix the support of the wave function for one particle. By Paulis principle the second particle must be located on a different point with respect to the first one. So we obtain the configuration space $\mathbb{C} \times \mathbb{C} \setminus D$ where D is the support of the other wave function. We denote this configuration space by C_2 . The many-particle case is similar and the N -fermion configuration space C_N has been constructed by the same procedure.

Now we consider the topology of C_N to describe all interactions of the many-particle system. In advance we note that the space $C_2 = \mathbb{C} \times \mathbb{C} \setminus D$ is homotopic to $\mathbb{C} \times \mathbb{C} \setminus \{w\}$ where w denotes the center coordinate of D . Obviously this must be true for C_N in the same sense. Let $(z_1, z_2, \dots, z_N) \in C_N$ with $z_i \in \mathbb{C}$ be the coordinate vector of the electrons. The guiding centers are denoted by $w_i \in \mathbb{C}$ for the i -th fermion. Because of the principle of identical particles we can change the order of the coordinates (z_1, z_2, \dots, z_N) without any effect on physical quantities. So finally we obtain the N -particle configuration space $B_N = C_N / S_N$ which is the above constructed space C_N apart from an arbitrary, symmetric permutation S_N of N -particles. Such spaces are extensively studied in the theory of knots and links [11], in the theory of iterated loop spaces [12] and in the singularity theory [13,14].

The set of complex, square-integrable functions over B_N forms the Fock space $F_N = L^2(B_N)$ of our system. We emphasize that according to [11] the topological description of magneto-electrons in F_N is generated by the holomorphic functions

$$f_N(z_1, \dots, z_N, w_1, \dots, w_N) = \sum_{k=1}^N \prod_{j=1}^k \frac{1}{(z_j - w_j)} = \sum_{k=1}^N f_{k,N}. \tag{29}$$

Thus we translate the topological information of B_N encoded in f_N into a suitable 2-dimensional manifold formed by the particles itself. As shown in [6] the torus T^2 is convenient to calculate the fractional filling factor. On the other hand the topological information to get out an effective Hamiltonian is given by the combinatorical structure of the triangulation. The simplices of such triangulation are equilateral triangles formed by three particles leading to a hexagonal configuration on the surface. Therefore we choose $N = 3$ and obtain the generator

$$\begin{aligned}
f_3(z_1, z_2, z_3, w_1, w_2, w_3) &= \frac{1}{z_1 - w_1} + \frac{1}{(z_1 - w_1)(z_2 - w_2)} + \\
&\quad + \frac{1}{(z_1 - w_1)(z_2 - w_2)(z_3 - w_3)}.
\end{aligned} \tag{30}$$

After the projection $B_3 \rightarrow T^2$ where all three coordinates z_1, z_2, z_3 will be projected on one coordinate $z \in T^2$ we get

$$\begin{aligned}
f_3(z, w_1, w_2, w_3) &= \frac{1}{z - w_1} + \frac{1}{(z - w_1)(z - w_2)} + \\
&\quad + \frac{1}{(z - w_1)(z - w_2)(z - w_3)}.
\end{aligned} \tag{31}$$

Although the projection loses some information about the topology of B_N encoded in (30) the main part is conserved in (31). For the interpretation of this generator we can only give an intuitive explanation derived from similar problems in conformal field theory (CFT). There is a rich literature about the relation between CFT and the FQHE (for instance [15,16]) including the relation to the Chern-Simons theory [17]. In [14] Arnold investigated the structure of interactions with singularity theory giving the answer to the question which functions generate the main properties of such systems. As an example we consider the set of harmonic functions including the Coulomb potential. According to the theorem of Hodge [18] this set is isomorphic to the cohomology groups of deRham [19,20]¹ the generators of which are interpretable as the interaction potentials. As an example of this principle we consider the case of a charged point particle over \mathbb{C} with charge density $\delta(z)$, $z \in \mathbb{C}$. From the viewpoint of physics we have to consider the Poisson equation

$$\Delta V(z) = \delta(z) \quad \text{in } \mathbb{C} \quad (32)$$

or we restrict the potential $V(z)$ to the set of harmonic functions given by the solutions of

$$\Delta \tilde{V}(z) = 0 \quad \text{in } \mathbb{C} \setminus \{0\} . \quad (33)$$

From the mathematical point of view all solutions of (33) are given by the Poincare dual of the generator of $H_1(\mathbb{C} \setminus \{0\}, \mathbb{Z}) = H_1(S^1, \mathbb{Z}) = \mathbb{Z}$. Thus if we consider the particles as holes of the space then the Poincare dual of the first homology group represents the (harmonic) potential.

In order to calculate the effective interaction potential for the concrete case of a triangular configuration we consider the barycenter coordinate z with $|z| = r/l_0$, $l_0 = l_0(B)$ and $\arg(z) = \phi$ of the triangle instead of the coordinates z_i , $i = 1, 2, 3, \dots$. Thus after the reduction to one triangle we obtain for the potential $V(z) = V(r, \phi)$ from (31) the following expression

$$V(z) = \frac{e}{l_0} \left[\frac{z^2 + z(1 - w_2 - w_3) + w_2 w_3 + 1}{(z - w_1)(z - w_2)(z - w_3)} \right] . \quad (34)$$

But in the equilateral triangle the distance between the barycenter and the points (w_1, w_2, w_3) is always equal and stationary. So we find the leading term

$$V(z) = \frac{e}{l_0} |z|^2 \exp(i\Delta\phi) . \quad (35)$$

The most important difference between our approach and the standard Coulomb interaction consists in the phase ϕ . The relevant phase difference $\Delta\phi = \pm 2\pi/3$ is given by the angles between lines connecting the barycenter with the vertex points. From the fact that the cohomology groups of the torus are generated by real functions we have to consider the real part of the interaction term which leads to

$$H_{eV} = \cos(2\pi/3) r^2 \Omega^2 = -\frac{1}{2} \Omega^2 r^2 \quad (36)$$

and justifies the expression (11) where the parameter $\omega_0^2(B)$ may be related to $\Omega^2(B)$. According to our calculation this three-body potential is harmonic and lifts degeneracy of the Landau levels.

The total ee-interaction may be written in the Form

$$H_{ee} = H_{eV} + W \quad (37)$$

where the unknown rest W describes many-body correlations beyond the short-range potential term (36). In section 3 this part of ee-interaction is phenomenologically treated by a cluster construction.

¹In ([21] p. 8) the cohomology of $\mathbb{R}^4 \setminus \mathbb{R}$ is calculated to state that the generator of the second cohomology group is given by the Coulomb potential.

V. SUMMARY AND CONCLUSIONS

In this paper we presented a new explanation of composite particles in the QHE theory. The crucial ee-interaction has been simulated by (i) a repulsive effective potential characterized by the parameter $\omega_0(B)$ responsible for primary CP and (ii) cluster formation of composite particles connected with absorption or emission of flux quanta. The role of collisions between electrons and randomly distributed impurities described by the parameter τ^* , consists in limitation of the existence of composite particles $\nu = \frac{p}{q}$ with higher denominators.

The experimentally relevant quasi-particles $\nu = \frac{p}{q}$, q - odd, are composite bosons transmuting to composite fermions with filling factors $\nu = \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \dots$ [9,10].

If the magnetic field B increases the number of flux quanta per electron also increases whereas the influence of ee-interaction decreases for primary CP. For bosonic CP ee-interaction is responsible both for formation and decomposition of clusters of primary CP. In consequence of the complicate interplay between eB- and ee-interactions the effective magnetic field B^* in the quantum liquid is expected to show characteristic minima [9].

The numerator in the filling factor $\nu = \frac{p}{q}$ can be interpreted both as number N of quasi-particles bounded in clusters or in the sense of $N = n + 1$ as the index of renormalized by $\hbar\omega_c \rightarrow \hbar\tilde{\omega}$ Landau levels included in electron correlation.

A geometric foundation of the phenomenological picture for N -body interactions under QHE conditions can be given by generation of holomorphic functions of the type (29). A topological interpretation of the generally fractional filling factor as a linking number has been given in [6]. In a recent paper [22] Fradkin et al. demonstrated how ee-interaction under QHE conditions may be discussed by a flux-attachment procedure within an effective field theory for Pfaffian states.

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energy levels splitted from the lowest renormalized Landau level	variation range of the ee-interaction parameter $\frac{\omega_c}{\tilde{\omega}}$
$\tilde{E}_{00} = \hbar\tilde{\omega} \left(\frac{1}{2} - 0 \right)$ LLL	$1 < \frac{\omega_c}{\tilde{\omega}} < 2 + \eta$, $0 < \eta \ll 1$
$\tilde{E}_{0,-1} = \hbar\tilde{\omega} \left(1 - \frac{1}{2} \frac{\omega_c}{\tilde{\omega}} \right)$	$1 < \frac{\omega_c}{\tilde{\omega}} < 2$
$\tilde{E}_{0,-2} = \hbar\tilde{\omega} \left(\frac{3}{2} - \frac{\omega_c}{\tilde{\omega}} \right)$	$1 < \frac{\omega_c}{\tilde{\omega}} < \frac{3}{2}$
$\tilde{E}_{0,-3} = \hbar\tilde{\omega} \left(2 - \frac{3}{2} \frac{\omega_c}{\tilde{\omega}} \right)$	$1 < \frac{\omega_c}{\tilde{\omega}} < \frac{4}{3}$
$\tilde{E}_{0,-4} = \hbar\tilde{\omega} \left(\frac{5}{2} - 2 \frac{\omega_c}{\tilde{\omega}} \right)$	$1 < \frac{\omega_c}{\tilde{\omega}} < \frac{5}{4}$
$\tilde{E}_{0,-5} = \hbar\tilde{\omega} \left(3 - \frac{5}{2} \frac{\omega_c}{\tilde{\omega}} \right)$	$1 < \frac{\omega_c}{\tilde{\omega}} < \frac{6}{5}$
\cdot	\cdot
\cdot	\cdot
\cdot	\cdot
$ m \longrightarrow \infty$	$\frac{\omega_c}{\tilde{\omega}} \longrightarrow 1^+$

TABLE I. quasi-particle energy levels for $n = 0$ and $m = 0, -1, -2, -3, -4, -5, \dots$